ADA ALGEBRAS

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Abstract. Let A be an artin algebra. We are interested in studying the representation theory of A, thus the category mod$A$ of finitely generated right A-modules. One of the classes of algebras whose representation theory is best understood is that of the quasi-tilted algebras introduced by Happel, Reiten and Smalø . In particular, the ideas and techniques introduced in this paper were used to define and study successfully several generalisations of quasi-tilted algebras, such as shod, weakly shod, laura, left or right supported algebras.

We introduce and study a new class, which we call ada algebras. This also generalises quasi-tilted algebras. Indeed, an artin algebra is quasi-tilted if and only if every indecomposable projective module lies in the so-called left part of the module category, a equivalently if and only if every indecomposable injective module lies in the right part. We say that an algebra is ada if any indecomposable projective and any indecomposable injective lies in the union of these two parts. Ada algebras have the nice property that their representation theory is entirely contained in that of two tilted algebras. Namely, the left support A_λ of an artin algebra is the endomorphism ring of the direct sum of all the indecomposable projective modules lying in the left part of mod$A$, and the right support A_ρ is defined dually. We prove that the left and right support of an ada algebra are tilted and describe the structure of the module category as in the following theorem.

Theorem A Let A be an ada algebra which is not quasi-tilted. There exists a finite family $(\Gamma_i)_{i=1}^t$ of Auslander-Reiten components of mod$A$ which are directed, generalised standard, convex and containing right sections such that:

(a) $indA = indA_\lambda \cup indA_\rho$ and each of A_λ and A_ρ is a direct product of tilted algebras.

(b) If Γ is an Auslander-Reiten component of mod$A$ distinct from the Γ_i, then Γ is an Auslander-Reiten component of either modA_λ or modA_ρ. Moreover

(i) If $Hom_A(\Gamma, \cup_i \Gamma_i) \neq 0$, then Γ is an Auslander-Reiten component of modA_λ, and,

(ii) If $Hom_A(\cup i \Gamma_i, \Gamma) \neq 0$, then Γ is an Auslander-Reiten component of modA_ρ.

Furthermore, the portion of the module category of an ada algebra which lies neither in the left nor in the right part is fairly well-understood.

Considering next the case where A is a finite dimensional algebra over an algebraically closed field, we study its simple connectedness. We recall that a triangular algebra A is called simply connected if the fundamental group of any bound quiver presentation of A is trivial. A well-known problem of Skowroński links the simple connectedness of A to the vanishing of the first Hochschild cohomology group $HH^1(A)$ of A with coefficients in the bimodule ${}_AA_A$. The equivalence of these conditions holds true for several classes of algebras, and among others for tilted algebras. This brings us to our second theorem.

Theorem B Let A be an ada algebra over an algebraically closed field. Then A is simply connected if and only if $HH^1(A) = 0$.

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